## MATH 1650 GRAPHS OF FUNCTIONS

**EXAMPLE:** Determine whether the following functions are even, odd, or neither even nor odd.
Check your answer graphically.

- $f(x) = 3x\sqrt{x^2-1}$

  $f(-x) = 3(-x)\sqrt{(-x)^2-1} = -3x\sqrt{x^2-1} = -f(x)$ so $f$ is odd.

  Graphing $y = 3x\sqrt{x^2-1}$ suggests the graph is symmetric about the origin.

- $g(x) = \dfrac{x^2+1}{x^2-1}$

  $g(-x) = \dfrac{(-x)^2+1}{(-x)^2-1} = \dfrac{x^2+1}{x^2-1} = g(x)$ so $g$ is even.

  Graphing $y = \dfrac{x^2+1}{x^2-1}$ suggests the graph is symmetric about the $y$-axis.

- $h(x) = 3x^2 - 2x + 1$.

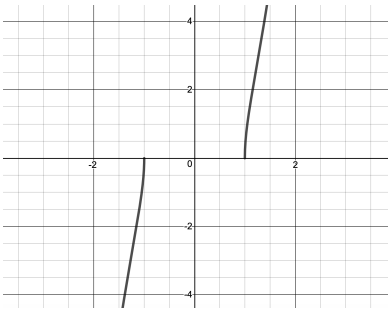
  $h(-x) = 3(-x)^2 - 2(-x) + 1 = 3x^2 + 2x + 1$ which is different than $h(x)$ so $h$ doesn't appear to be even.

  To prove $h$ isn't even, we find: $h(1) = 3(1)^2 - 2(1) + 1 = 2$ and $h(-1) = 3(-1)^2 - 2(-1) + 1 = 6$.

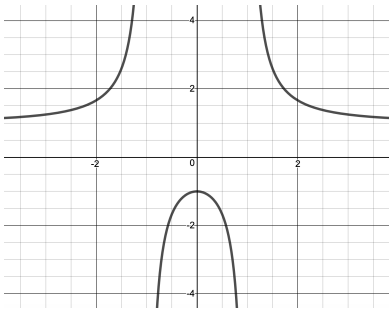
  Since $h(-1) \neq h(1)$, $h$ is not even.

  As a bonus, since $h(-1) \neq -h(1)$, we know $h$ cannot be odd, either.

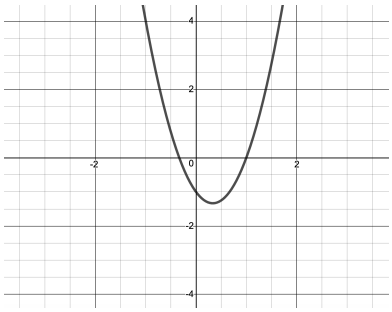
  Graphing $y = 3x^2 - 2x + 1$, the graph doesn't appear to be symmetric about the $y$-axis nor the origin.




$y = f(x)$



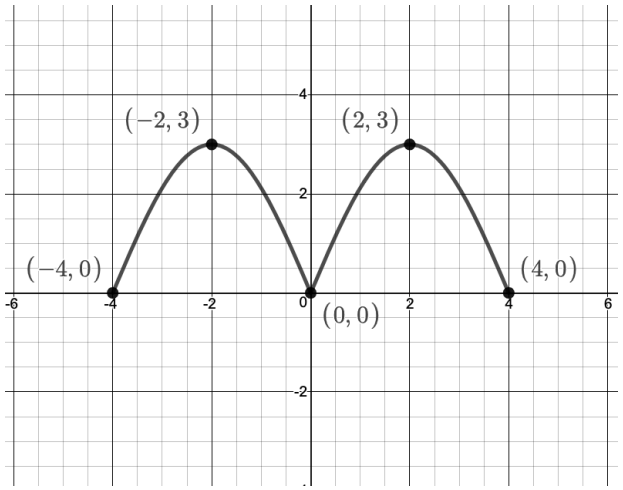

$y = g(x)$



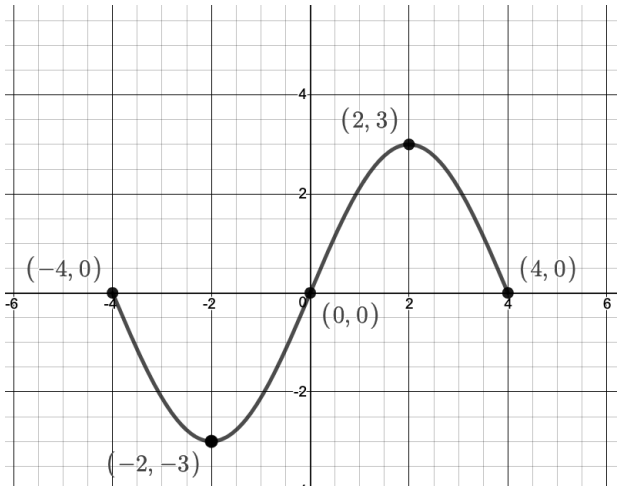

$y = h(x)$

**EXAMPLE:** Below is the (partial) graph of a function, $f$. Complete each graph as directed:

Complete the graph assuming $f$ is **even**.




Complete the graph assuming $f$ is **odd**.




**EXAMPLE:** Answer the following questions using the graph of $f$:

- Domain: $[-4, 6)$
- Range: $[-8, 5.5)$
- $f(-2) = 4.5$
- $(0, 0)$ is the $y$-intercept.
- Zeros of $f$: $x \approx -3.5, 0, 5.5$
- $f(x) = -6$: $\{1.5\} \cup [4, 5]$.
- $f$ is increasing on:

  $[-4, -2] \cup [3, 4] \cup [5, 6)$.
- $f$ is decreasing on:

  $[-2, 3]$
- $f$ is constant on:

  $[4, 5]$
- Local maximums:

  $(-2, 4.5)$, $\{(x, -6) \mid 4 \leq x < 5\}$
- Local minimums:

  $(3, -8)$, $\{(x, -6) \mid 4 < x \leq 5\}$
- The minimum is $-8$.

  $[4, 5]$

$f$ has no maximum since the point $(6, 5.5)$ is not included on the graph.